Probability and Random Processes

ECS 315

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Summary

Office Hours:

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Wednesday	14:00-15:30
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Chapter 7

- RV = Random Variable
  - Use capital letters.
- Defining event from a statement about a RV:
$$[X > 7] \equiv \{\omega : X(\omega) > 7\}$$
- Probability of an event involving a RV
$$P[X > 7] \equiv P([X > 7]) \equiv P(\{\omega : X(\omega) > 7\})$$
- In general, the **support** of a RV X is any set S such that
$$P[X \in S] = 1.$$
- A RV is discrete iff it has a countable support.

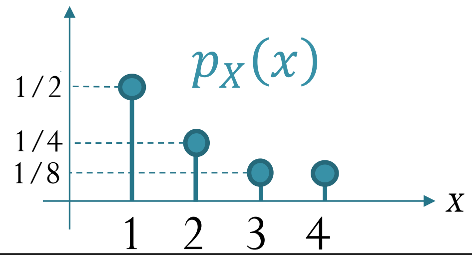
Sections 8.1-8.2

- (Default) **support** for discrete RV: $S_X = \{x: p_X(x) > 0\}$

- **Probability Mass Function (pmf)**:

  $$p_X(x) \equiv P[X = x]$$

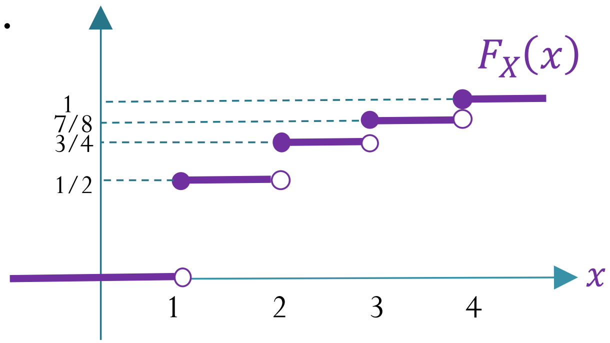
  - Useful only for discrete RVs.
  - Defined for all $x \in \mathbb{R}$
    - "0, otherwise" in its description.
  - **Distribution**
  - Two characterizing properties:
    - $p_X(x) \geq 0$
    - $\sum_x p_X(x) = 1$
  - Stem plot



- **Cumulative Distribution Function (cdf)**

  $$F_X(x) \equiv P[X \leq x]$$

  - Useful for any kind of RVs.
  - Defined for all $x \in \mathbb{R}$
  - Three characterizing properties:
    1. Non-decreasing
    2. Right-continuous
    3. $\lim_{x \to -\infty} F_X(x) = 0$ and $\lim_{x \to \infty} F_X(x) = 1$.
  - For discrete RV: Staircase function with jumps whose size at $x = c$ gives $P[X = c]$.



Section 8.3

$X \sim$	Support S_X	$p_X(x) =$
Uniform $\mathcal{U}(S)$	S	$\begin{cases} \frac{1}{\lvert S \rvert}, & x \in S, \\ 0, & \text{otherwise.} \end{cases}$
Bernoulli(p) ↑ $n = 1$	$\{0, 1\}$	$\begin{cases} 1 - p, & x = 0, \\ p, & x = 1, \\ 0, & \text{otherwise.} \end{cases}$
Binomial $\mathcal{B}(n, p)$	$\{0, 1, \ldots, n\}$	$\begin{cases} \binom{n}{x} p^x (1-p)^{n-x}, & x = 0, 1, 2, \ldots, n, \\ 0, & \text{otherwise.} \end{cases}$
Geometric $\mathcal{G}_0(p)$	$\mathbb{N} \cup \{0\}$	$\begin{cases} p(1-p)^x, & x = 0, 1, 2, \ldots, \\ 0, & \text{otherwise.} \end{cases}$
Geometric $\mathcal{G}_1(p)$	$\mathbb{N}$	$\begin{cases} p(1-p)^{x-1}, & x = 1, 2, \ldots, \\ 0, & \text{otherwise.} \end{cases}$
Poisson $\mathcal{P}(\alpha)$	$\mathbb{N} \cup \{0\}$	$\begin{cases} e^{-\alpha} \frac{\alpha^x}{x!}, & x = 0, 1, 2, \ldots, \\ 0, & \text{otherwise} \end{cases}$

Ch. 9: Expectation and Variance

- The **expectation** (or **mean** or **expected value**) of a discrete random variable X is given by

$$\mathbb{E}X = \sum_x x p_X(x)$$

- The expected value of a function g of a RV X is given by

$$\mathbb{E}\left[g(X)\right] = \sum_x g(x) p_X(x)$$

- The **variance** of a RV X is given by

$$\mathrm{Var}\left[X\right] = \mathbb{E}\left[\left(X - \mathbb{E}X\right)^2\right] = \mathbb{E}\left[X^2\right] - \left(\mathbb{E}X\right)^2$$

- The **standard deviation** of a RV X is given by

$$\sigma_X = \sqrt{\mathrm{Var}\left[X\right]}$$